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Heat transfer and friction of a rough cylinder in longitudinal turbulent gas flow with variable physical properties

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Abstract—In the cylindrical working section of a high-pressure wind tunnel an experimental investigation is made of heat transfer and friction of a heat generating cylinder with rectangular roughness in longitudinal turbulent gas flow in the following ranges: $Re_x = 6\times10^4$–3×10^7, $Re_{r_0} = 2\times10^4$–2×10^5, $k^+ = 5$–200 and $\psi = T_w/T_f = 1$–3. Boundary layer velocity and temperature profiles are measured, their deformation with an increase in ψ and k^+ is shown, as well as the universality of the velocity defect law and of the hydrodynamic function of roughness, relations are presented for finding the momentum thickness δ^{**} in the boundary layer of a rough cylinder. Formulae are obtained for engineering calculations of heat transfer and hydraulic resistance that interpolate the entire region of transition from the regime without the effect of roughness to that with a fully effective roughness and that take into account the variability of the physical properties of gas with an increase in the temperature factor up to $\psi = 3$.

1. INTRODUCTION

FOR A NUMBER of years now, at the Institute of Physical and Technical Problems of Energetics of the Lithuanian Academy of Sciences, investigations have been carried out into heat transfer and hydraulic resistance in rough gas-cooled annular channels at high heat fluxes as applied to the enhancement of heat transfer in fuel assemblies of nuclear reactors. The results of these investigations enabled correlations that take into account the effect of the temperature factor and of the height of roughness elements of heat transfer and friction in wide ranges of operational parameters and heights of roughness elements to be made [1]. It remains to be learned, however, concerning the physical reasons that account for the heat transfer rate in each specific case. For these reasons to be elucidated, it was necessary to first study velocity and temperature fields near a heat generating rough surface. To this end, it was decided to measure these quantities on a rough cylinder in longitudinal flow in a cylindrical working section of a high-pressure wind tunnel, because a large complex of investigations was made earlier under these conditions with a smooth cylinder in the presence of great temperature differences in a boundary layer [2–6].

Moreover, up to now there have been no investigations of velocity and temperature fields and of heat transfer in boundary layers formed on rough surfaces in longitudinal flow when the variability of physical properties shows up most appreciably in a gas flow. Therefore, these investigations are of scientific interest for their own sake for understanding the mechanism of transfer in strongly non-isothermal boundary layers.

The objectives of this work were to present experimental results for velocity and temperature profiles in a boundary layer, to carry out their processing and analysis, to obtain reliable relations for engineering calculations of local heat transfer and friction of a heat generating rough cylinder in a longitudinal gas flow with variable physical properties within the following ranges: $Re_x = 6\times10^4$–3×10^7, $Re_{r_0} = 2\times10^4$–2×10^5, $k^+ = 5$–200 and $\psi = T_w/T_f = 1$–3.

2. EXPERIMENTAL PROCEDURE

Investigations were carried out in the initial section of a vertical annular channel when boundary layers, which develop on the outer smooth wind and on the rough cylinder made of a 16 mm diameter tubing, did not converge to the exit from a 1 m long test section. The shape of roughness on the cylinder was rectangular, the height of roughness elements was $k = 0.13$ mm and the pitch $s = 1$ mm. Even though only one rough cylinder was investigated, not only a wide range of Re_{r_0} was obtained, but also the variation of k^+ from 5 to 200, since the test section was connected to a closed aerodynamic loop of high pressure which varied from 0.1 to 2 MPa.

In the boundary layer of the rough cylinder the measurements were made at distances of 125, 325, 525 and 825 mm from the beginning of the cylinder. A thermal boundary layer on the cylinder developed simultaneously with the hydrodynamic one. Velocity profiles were measured by Pitot–Prandtl tubes. To measure the minimum thickness, a 0.8 mm diameter Pitot tube was flattened out so that the thickness of the rectangular entrance slit amounted to 0.1 mm.

NOMENCLATURE

- B constant in the velocity defect law
- c constant in the heat transfer law
- c_f local friction coefficient, $2\tau_w/\rho U^2$
- c_p specific heat at constant pressure
- D constant in equation for velocity profile displacement
- d diameter
- k height of roughness elements
- k^+ dimensionless height, ku_*/ν
- m exponent on Re_x in heat transfer law
- n exponent on ψ
- q heat flux density
- $R(k^+)$ hydrodynamic function of roughness
- r radial distance from rotation axis
- r_0 volumetric radius of cylinder
- r_0^+ dimensionless radius, $r_0 u_*/\nu$
- s pitch of roughness elements
- T temperature
- Tu level of turbulence, $(\overline{u'^2})^{1/2}/U$
- U free stream velocity
- u mean longitudinal velocity
- U^+, u^+ dimensionless velocities, U/u_* and u/u_*, respectively
- u_* dynamic velocity, $(\tau_w/\rho)^{1/2}$
- u' longitudinal velocity fluctuation
- x distance from cylinder leading edge
- y distance over the radius reckoned from tips of roughness elements
- y_R coordinate of axisymmetric boundary layer, $r_0 \ln(r/r_0)$
- y_R^+ dimensionless coordinate, $r_0^+ \ln(r/r_0)$.

Greek symbols

- α heat transfer coefficient
- δ hydrodynamic boundary layer thickness
- δ^* displacement thickness,

$$r_0\left(\sqrt{\left(1+(2/r_0)\times\int_0^\infty (1-(\rho u/\rho_f U)(r/r_0)\,dy-1\right)}\right)$$

- δ^{**} momentum thickness,

$$r_0\left(\sqrt{\left(1+(2/r_0)\times\int_0^\infty (U-u/U)(\rho u/\rho_f U)(r/r_0)\,dy-1\right)}\right)$$

- θ temperature difference, $T_w - T$
- θ_* characteristic temperature, $q_w/\rho c_p u_*$
- θ^+ dimensionless temperature, θ/θ_*
- κ von Karman constant, 0.41
- λ thermal conductivity
- μ dynamic viscosity
- ν kinematic viscosity
- ρ density
- τ shear stress
- ψ temperature factor, T_w/T_f.

Dimensionless symbols

- Nu local Nusselt number, $\alpha x/\lambda$
- Re_{r_0} Reynolds number, $r_0 U/\nu$
- Re_x local Reynolds number, xU/ν
- Re_{δ^*} Reynolds number based on displacement thickness, $\delta^* U/\nu$
- St Stanton number, $\alpha/\rho c_p U$ or $1/U^+\theta_\infty^+$.

Subscripts

- f, ∞ free stream
- s sand roughness
- sm smooth surface
- w wall
- ψ variable physical properties
- $\psi = 1$ constant physical properties.

Pressure drops were measured by inductive transducers while the entire measuring information was introduced directly into an electronic computer. The translation of transducers across the boundary layer was made with the aid of traversing mechanisms with micrometer screws. The initial position of transducers was determined by electric contact with the tips of roughness elements. For more accurate adjustment and fixing of the sensitive elements of a probe relative to the axisymmetric surface of the rough cylinder, the probe was developed and patented with a Π-like plug fitted the legs of which slid along the cylinder side surface [7].

Temperature profiles were measured by microthermocouples. To measure the wall temperature, 14 thermocouples were welded to the inner side of the cylinder. These thermocouples were also used to measure the change of voltage along the cylinder. The measurement of all electrical signals from the thermocouples and also the measurement of the voltage changes on the cylinder and shunts were made with the aid of an automatic data acquisition measuring system. Heat generation in the cylinder was provided by passing a constant electric current through it (q_w = const.). The high stability of the generated voltage and, consequently, the stability of heat generation in the cylinder were attained by a special electronic stabilizing device.

The external flow turbulence (Tu_∞) at the entrance to the test section amounted to 0.9–1.1% depending on air pressure in the loop and somewhat increased downstream attaining 1.6–1.8% at the exit.

To determine the surface friction coefficient c_f, when the velocity distribution in an isothermal boundary layer is known, use is often made of the universal velocity defect law

$$U^+ - u^+ = -\frac{1}{\kappa}\ln\frac{y}{\delta^* U^+} + B. \qquad (1)$$

In the absence of the free stream flow perturbations $B = -0.8$ for a smooth and a rough plate [8]. The substitution of y in equation (1) by the coordinate

$$y_R = r_0 \ln (r/r_0) \qquad (2)$$

first suggested by Rao [9], allows the logarithmic excess velocity distribution in an axisymmetric boundary layer to be expressed by a universal relation for a plate [2], but B varies with a change in the transverse curvature of a smooth surface, because this quantity depends on the ratio δ^*/r_0 and, for a particular value of r_0, on Re_{δ^*} [4].

In the present work c_f and the dependence of B on the transverse curvature of a rough surface in an isothermal boundary layer are found from the measured velocity profiles with the aid of the transformed relation (1)

$$1 - \frac{u}{U} = -\frac{1}{\kappa}\sqrt{\left(\frac{c_f}{2}\right)}\log\left(\frac{r_0 \ln (r/r_0)}{\delta^*}\sqrt{\left(\frac{c_f}{2}\right)}\right) + B\sqrt{\left(\frac{c_f}{2}\right)}. \qquad (3)$$

By employing the iteration technique such values of c_f and B were selected, which would satisfy relation (3) for all the measured values of the logarithmic part in the velocity profile. To this end, the program for identifying the optimum coefficients by the techniques of a deformed polygon and random sampling was used.

The validity of the velocity defect law (1) on a plane rough surface depends on the start of reckoning of the coordinate y. The parameter, which would determine the flow and heat transfer, is to be a certain effective vortical layer thickness near a rough wall. This will reduce to a minimum the effect of the shape of the roughness elements. Therefore, a common particle for a plane surface uses the well-known idea about such a displacement of the y-coordinate origin from the tip of roughness elements to the wall at which the von Karman constant in relation (1) remains equal to $\kappa = 0.41$. The use of coordinate (2), which involves a volumetric radius of the rough cylinder r_0, automatically, to some extent, displaces the origin of the coordinate y_R^+ to the wall. In the present case, the cylinder radius drawn through the tips of roughness elements is equal to 7.96 mm and the volumetric radius r_0 is equal to 7.84 mm.

For determining c_f under the conditions of variable physical properties, simultaneous measurements were made of local heat transfer and velocity/temperature profiles in the given section of a boundary layer. Since $St = 1/U^+\theta_\infty^+$, such a value of $u_{*\psi}$ was selected by the iteration technique at which the values of St, obtained from measurements of local heat transfer and velocity/temperature profiles, were the same.

The problem of determining, analysis and correlation of data on the local heat transfer of a rough cylinder in longitudinal flow with variable physical properties was complicated by the overlapping effects of three factors: transverse curvature of the surface, its roughness and temperature factor ψ. The influence of the transverse curvature of the surface depends, for instance, on δ^*/r_0 and, in turn, $\delta^* = f(Re_x, Re_{r_0}, x/k, Tu_\infty, \psi)$. Even a minor error in the determination of $Nu_{\psi=1}$ for $Nu/Nu_{\psi=1} = f(\psi)$ will immediately affect $f(\psi)$, while it is impossible to directly perform accurate calculations of the scale quantity $Nu_{\psi=1}$ from experimental results without taking into account the influence of the above-mentioned factors.

The aforegoing considerations have predetermined the procedures of experiments and data processing. All the experiments were carried out at q_w = const. in series with 6–32 regimes in each. In some series, a certain constant temperature T_w was maintained in the given section and the regimes differed by the values of Re_{r_0} (at r_0 = const.) and by the magnitude of the heat flux. In other series Re_{r_0} was held constant, while the regimes differed by T_w in the given section (in each 10 K up to T_w = 450 K and further on in 25 K up to 1000 K) and by the magnitude of the heat flux. This procedure made it possible not only to carry out investigations with optimum (as regards the accuracy of experiments) relationships between operational parameters, but also to obtain the data in the form most convenient for analysis and correlation.

3. VELOCITY AND TEMPERATURE PROFILES AND FRICTION

It is found by measurements that the values of δ^*/r_0 on a rough cylinder are several times higher than on a smooth one under the same conditions, whereas constant B in the velocity defect law (1) depends on δ^*/r_0 [4]. However, under isothermal conditions at δ^*/r_0 = const. and in the case of the use of coordinate (2) the universality of the velocity defect law for a smooth cylinder is also preserved for a rough one irrespective of the values of k^+ (Fig. 1). The values of B obtained in the present work for equation (3) for all the regimes with effective roughness are correlated by the relation

$$-B = 2.2\delta^*/r_0 + 1.7. \qquad (4)$$

The effect of additional shear stresses brought about

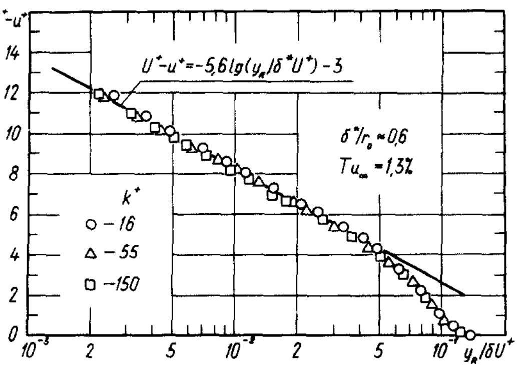

FIG. 1. Excess velocity distribution in a boundary layer of a rough cylinder at different k^+ but constant δ/r_0 and Tu_∞: $x/k = 4\times10^3$.

by roughness elements can be inferred from velocity profiles measured under isothermal conditions and presented in universal coordinates of the wall law (Fig. 2). In the logarithmic zone the von Karman constant remains intact, while the displacement of the velocity profiles Δu^+ depends on the roughness parameter k^+ which takes into account the relationship between pressure forces in the vicinity of roughness elements and viscous forces in the overall quantity τ_Σ, which corresponds to the quantity τ_w for a smooth wall.

Velocity profiles on a smooth and a rough cylinder, presented in Fig. 2, were obtained approximately at the same free stream turbulence ($Tu_\infty \approx 1.6$–1.8%). It can be clearly seen that Tu_∞ exerts a much smaller influence on velocity profiles in the outer region of the boundary layer than on a smooth cylinder. It should be noted that a boundary layer on a rough cylinder is several times thicker than on a smooth one under the same conditions.

For the studied type of roughness under isothermal conditions the displacement of velocity profiles Δu^+ is much more appreciable than for a sand roughness (Fig. 3), but in regimes with a fully effective roughness ($k^+ > 70$), when the greater portion of resistance is provided by the shape of separate roughness elements, the resistance law turns to be purely quadratic (Fig. 4), therefore the displacement of velocity profiles in this zone obeys the logarithmic relation

$$\frac{\Delta u}{u_*} = 5.6\log k^+ + D \tag{5}$$

where for the present roughness $D = -1.4$. To this value of D there corresponds an equivalent sand roughness $k_s = 2.47k$. In the wall law for a rough axisymmetric surface

$$u^+ = 5.6\log(y_R/k) + R(k^+) \tag{6}$$

the hydrodynamic function of roughness $R(k^+)$ is equal to $4.9 - D = 6.3$ for the roughness studied and is independent of the effect of the transverse surface curvature (Fig. 5). Many researchers, who carried out investigations in annular channels [10, 11], employed the inadequate wall law for the tube after the 'separation of zones' for correlating the velocity profile of an inner cylinder and obtained a great variety of values of κ and $R(k^+)$ in relation (6). The application of coordinate (2) enables the universality of this relation to be preserved for flow past a rough cylinder also (Fig. 5).

In regimes with a fully effective roughness, studied in the present work, the values of c_f were correlated by the relation

$$\sqrt{\left(\frac{c_f}{2}\right)} = 5.6\log\frac{Re_{\delta^*}}{k^+} - 2.2\delta^*/r_0 + 4.6. \tag{7}$$

In order to track the deformation of velocity and temperature profiles on a rough cylinder in the case of variable physical properties of a gas and compare it with a similar deformation on a smooth cylinder, it is necessary to consider either mass velocity profiles (Fig. 6), or velocity and temperature profiles in universal coordinates where the values of physical par-

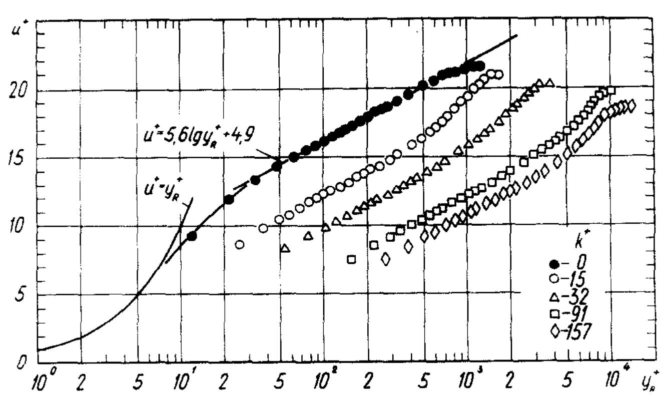

FIG. 2. The universal velocity distribution law for turbulent flow past a smooth and a rough cylinder: $x/k = 6.3\times10^3$, $Tu_\infty \approx 1.6\%$.

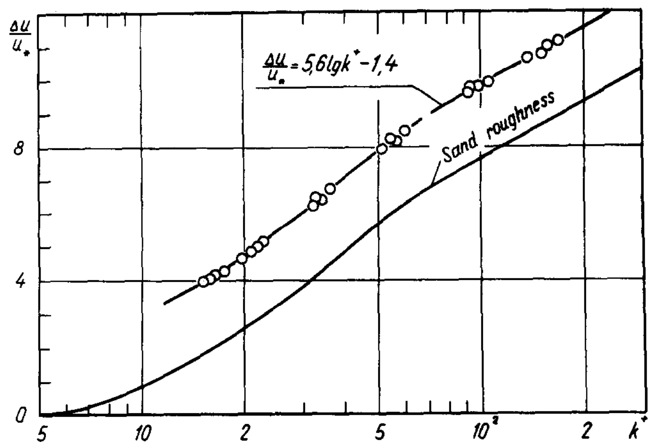

FIG. 3. Displacement in velocity profiles of a rough cylinder with respect to velocity profiles of a smooth cylinder in universal coordinates vs k^+.

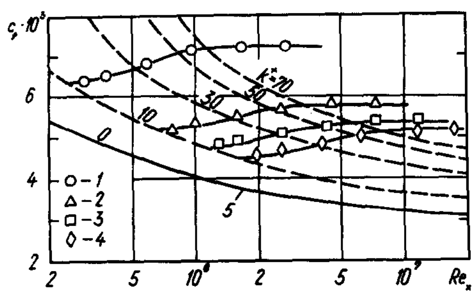

FIG. 4. The law of resistance of a rough cylinder; local friction coefficient: 1–4, $x/k = 9.6 \times 10^2$, 2.5×10^3, 4×10^3 and 6.3×10^3, respectively; 5, hydraulically smooth cylinder at $Re_{r_0} = 2 \times 10^4$.

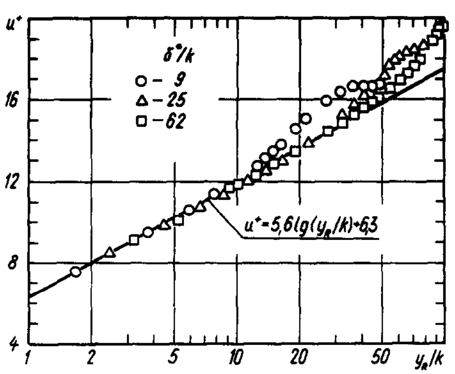

FIG. 5. Universal distribution of velocity and determination of the hydrodynamic function of roughness $R(k^+)$ in the regime with a fully effective roughness: $k^+ = 140$–170, $Re_{r_0} = 1.7 \times 10^5$.

ameters, which are incorporated into the quantities u^+, θ^+ and y_R^+, are based on T_f (Fig. 7).

With an increase of the temperature factor ψ in the regime of a partially effective roughness (Fig. 6(b)) the mass velocity profile undergoes almost the same or only somewhat weaker deformation as on a smooth surface (Fig. 6(a)), while in the regime of a fully effective roughness (Fig. 6(d)) a much weaker deformation than on a smooth surface (Fig. 6(c)).

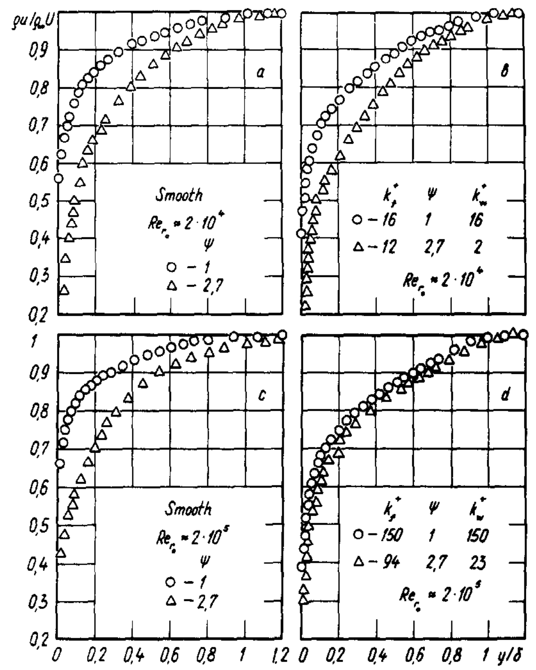

FIG. 6. Deformation of mass velocity profiles with an increase of the temperature factor on a hydraulically smooth cylinder (a, c) and a rough cylinder in the transient regime (b) and in the regime with a fully effective roughness (d).

It is seen from velocity and temperature profiles in universal coordinates (Fig. 7) that on a smooth surface in an isothermal flow the effect of viscosity is restricted by the region of a viscous sublayer and transition zone, and with an increase of ψ its effect is exhibited farther and farther from the wall with the extension of the zone of transition. On a rough cylinder the velocity profile undergoes a much stronger deformation with an increase of ψ. The temperature profiles of a rough cylinder in flow do not show the wake effect (Fig. 7).

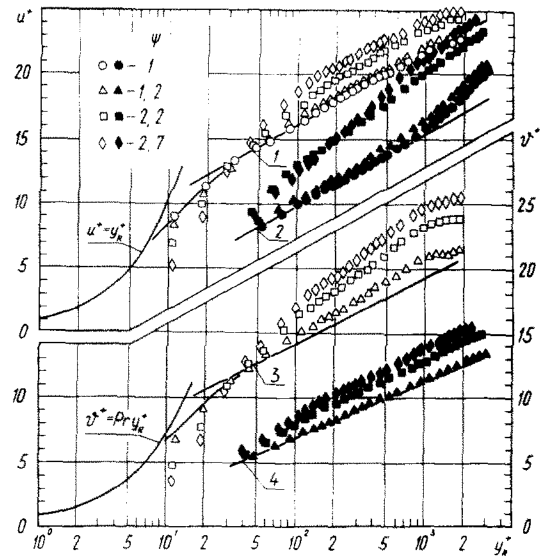

FIG. 7. Deformation of velocity and temperature profiles with an increase of ψ on smooth (light symbols) and rough (dark symbols) cylinders in the wall coordinates: 1, $k^+ = 0$, $u^+ = 5.6 \log y_R^+ + 4.9$; 2, $k^+ = 32$, $u^+ = 5.6 \log y_R^+ - 1.4$; 3, $k^+ = 0$, $\theta^+ = 5.1 \log y_R^+ + 3.8$; 4, $k^+ = 32$, $\theta^+ = 4.2 \log y_R^+ - 1.6$.

However, when the determining parameters are taken into account the variability of physical properties and of the change in the mass velocity across the boundary layer, then heat transfer and friction can be correlated by the same relations as those used for constant physical properties. In regimes with a fully effective roughness the quantity c_f at variable physical properties is correlated by relation (7) when k^+ is replaced by the parameter $k_w^+ = ku_{*\psi}/\nu_w$ in which the physical properties are governed by T_w, i.e. a reduced momentum transfer is taken into account because of the change in μ and ρ. In equation (7) the determining quantity is taken to be the local integral characteristic of the boundary layer which incorporates a change in the mass velocity across the boundary layer—the displacement thickness δ^*. The variability of physical properties does not exert identical effects on the integral thicknesses of the boundary layer on both a smooth and a rough cylinder. The displacement thickness increases with ψ by the thickness of enthalpy increase. It is found by the present measurements of velocity and temperature fields that with an increase of ψ from 1 to 3 the quantity δ^* on a rough cylinder increases by about a factor of 2. At the same time the value of δ^{**} virtually does not alter with an increase of ψ. Therefore, when δ^{**} is used as the determining one, it is necessary to account separately for the effect of the variability of physical properties, while the use of δ^* makes it unnecessary to introduce ψ into dimensionless equations. The experimental data on the resistance of a smooth cylinder at constant and variable physical properties [4] can be correlated as

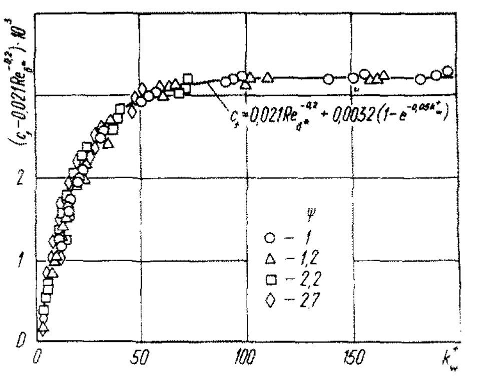

FIG. 8. An increase in the local coefficient of friction of a cylinder with an increase of k_w^+ at constant and variable physical properties of a heat transfer agent.

$$c_f = 0.021 Re_{\delta^*}^{-0.2}. \quad (8)$$

A formula has been derived for c_f which interpolates well the entire region of transition from the regime without the effect of roughness to the regime with a fully effective roughness. This formula, which has the form

$$c_f = 0.021 Re_{\delta^*}^{-0.2} + 0.0032[1 - \exp(-0.05 k_w^+)] \quad (9)$$

correlates the data obtained in the present work for the resistance of a rough cylinder at both constant and variable physical properties of a gas flow (Fig. 8). When $k_w^+ \to 0$, this formula passes over into equation (8) for a hydraulically smooth cylinder, and when $k_w^+ \to \infty$, into the formula for a fully rough cylinder. In the transition region equation (9) gives the curves of the 'technical roughness' type for the dependence of c_f on Re_x and x/k (Fig. 4).

4. HEAT TRANSFER

Experimental data obtained in the present work for heat transfer of a cylinder in longitudinal flow taking into account the variability of physical properties can be correlated by the formula of the same type as that for c_f which interpolates the entire region of transition from the regime without the effect of roughness to the regime with a fully effective roughness

$$St = 0.0146 Re_{\delta^*}^{-0.24} + 0.0025[1 - \exp(-0.07 k_w^+)]. \quad (10)$$

However, for engineering calculations it is more convenient to use dimensionless relations in which heat transfer is the function of such parameters as Re_x, Re_{r_0}, x/k, and ψ. For a flow in a rough tube an important role is played by the relative roughness k/r, where r is the tube radius. The analogue of this quantity for flow past a rough cylinder is the ratio k/δ, where δ is the boundary layer thickness. The essential difference between both flows is that for a tube the relative roughness k/r at a constant k remains constant, while for a cylinder the relative roughness k/δ decreases with distance from the leading edge, since δ increases downstream. It will be assumed for simplicity that the boundary layer becomes turbulent starting from the leading edge of the cylinder. Then, in the forward portion of the cylinder, where the ratio k/δ is great, there will be a certain portion with the regime of a fully effective roughness. It will be followed by a so-called region of transition and, finally, only when the cylinder is sufficiently long, by a portion without the effect of roughness. The boundaries between the above-mentioned sections are governed by the values of k^+ in the same way as in flows in rough tubes. Different regimes of the effect of roughness can also be obtained by varying u or, as it was done in the present experiments, by varying ν. Therefore, to correlate the data on heat transfer for longitudinal flow past both a rough plate and a rough cylinder, the parameters Re_x and k^+ are needed and it is also necessary to take into account the quantity δ. For example, in the present experiments Nu at $k^+ = 20$ and $Re_{r_0} = 2\times10^4$ is equal to Nu at $k^+ = 160$ and $Re_{r_0} = 2\times10^5$, since at the same values of Re_x, but different values of x in the first case $\delta \approx 27$ mm and in the second case $\delta \approx 6$ mm. Since δ can be found only approximately from experimental data and the virtual start of the turbulent boundary layer does not coincide with the leading edge of the cylinder, and δ^* increases rapidly with ψ, then, to correlate the data on local heat transfer of a rough cylinder, use was made in the present work of the ratio k/δ^{**} in the considered section of the boundary layer. For δ^{**} the following relation was obtained:

$$\delta^{**} = \delta^{**}_{sm} + 0.27kx/r_0 \tag{11}$$

where

$$\delta^{**}_{sm} = (0.036+0.03(1-e^{-30Tu_\infty}))x \times Re_x^{-0.2}(1+x/r_0)^{-0.2} \tag{12}$$

which at $k = 0$, $r_0 = \infty$ and $Tu_\infty = 0$ passes over into the formula of Schlichting [12] for a smooth plate at $Tu_\infty = 0$.

The effect of variable physical properties of gas on heat transfer was taken into account by the temperature factor in the form of the relation

$$Nu/Nu_{\psi=1} = f(\psi) \tag{13}$$

where the quantity $Nu_{\psi=1}$ was calculated from the power-law relation

$$Nu_{\psi=1} = c_{\psi=1}\, Re_x^m \tag{14}$$

while the constant $c_{\psi=1}$ for each Re_{r_0} and x/r_0 was determined experimentally, by extrapolating experimental data presented in the form

$$c = Nu/Re_x^m = f(\psi)$$

up to $\psi = 1$. The exponents m on Re_x were determined from experimental data obtained at small values of ψ (Fig. 9).

In the region with $\psi \leqslant 3$ it is possible to approximate the experimental data for $Nu/Nu_{\psi=1}$ by the power-law relation (Fig. 10) and to determine the exponents n on ψ.

The effect of variable physical properties of gas on heat transfer along the length of a rough cylinder increases, just as for a smooth cylinder [3], up to a certain, for each Re_{r_0}, stabilized value (Fig. 11). The exponent n on ψ increases sharply over the initial stretch, i.e. with an increase of the thermal boundary layer thickness, however, already at $x/r_0 \approx 40$ this increase becomes inappreciable. For the exponent n on ψ the following relation was obtained:

$$n = -3.5Re_{r_0}^{-0.2}[1-\exp(-0.1x/r_0)]. \tag{15}$$

A decrease of n on ψ with an increasing Re_{r_0} is attributed to an increasing flow turbulization in the wall layer as also testified by the increase of m on Re_x in the heat transfer law with x/r_0 (Fig. 9). The data obtained in the present work agree well with the experimental data for a developed flow in rough annular channels [1].

For engineering calculations of the local heat transfer of a cylinder in the longitudinal flow under the conditions with appreciably variable physical properties of a gas flow the following formula is suggested which interpolates the entire region of transition from the regime without the effect of roughness to the regime with a fully effective roughness:

$$Nu = (Nu_{sm} + 0.0032Re_{r_0}(k/\delta^{**})^{-1.1})\psi^n \tag{16}$$

where

$$Nu_{sm} = c\,Re_x^m(1+\delta^{**}_{sm}/r_0)^{1.3}(1+0.4(1-e^{-24Tu_\infty}))$$

$$c = 0.023,\quad m = 0.8 \quad \text{for } 6\times10^4 \leqslant Re_x \leqslant 2\times10^6$$

$$c = 0.0147,\quad m = 0.83 \quad \text{for } 2\times10^6 \leqslant Re_x \leqslant 3\times10^7$$

δ^{**}, δ^{**}_{sm} and n are determined from relations (11), (12) and (15), respectively.

Equation (16) correlates the present experimental data within $\pm5\%$ in the ranges $Re_x = 6\times10^4$–3×10^7, $Re_{r_0} = 2\times10^4$–2×10^5, $k^+ = 5$–200 and $\psi = 1$–3.

5. CONCLUSIONS

(1) In a cylindrical working section of a high-pressure wind tunnel, an experimental investigation is made of heat transfer, friction, velocity and temperature profiles of a heat generating cylinder, having the roughness of a rectangular shape and the pitch which is close to the optimum one from the viewpoint of heat transfer augmentation ($s/k \approx 8$), in longitudinal turbulent gas flow under the conditions of a strong effect of variable physical properties. By varying the pressure in a wind tunnel, a change in k^+ from 5 to 200 and in Re_{r_0} from 2×10^4 to 2×10^5 is obtained.

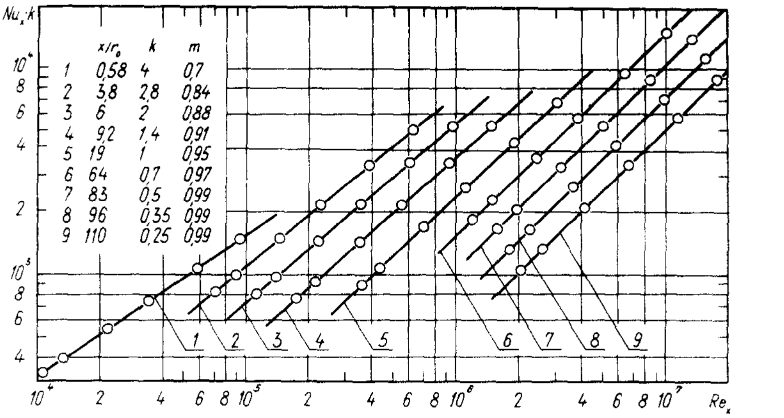

FIG. 9. Values of the exponent m in the relation $Nu = c\ Re_x^m$ at different x/r_0: $\psi \leq 1.2$.

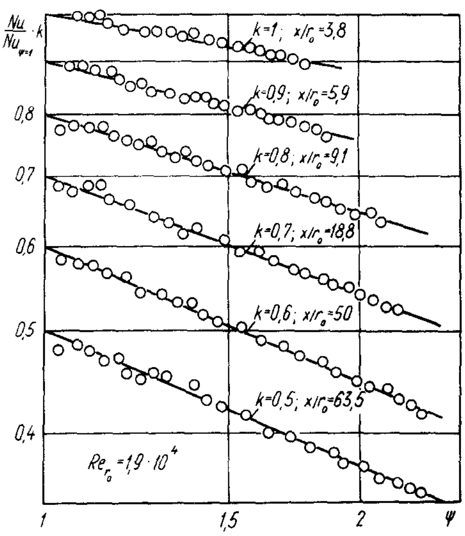

FIG. 10. The effect of the temperature factor on heat transfer of a rough cylinder at different x/r_0.

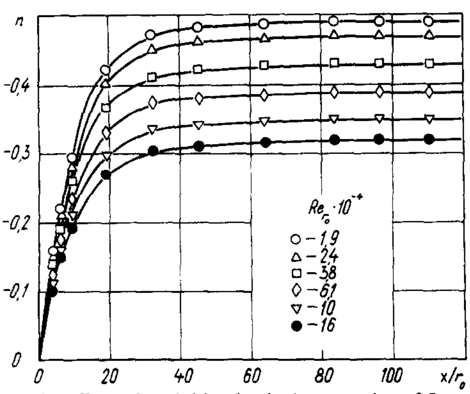

FIG. 11. The effect of variable physical properties of flow on heat transfer vs x/r_0 at different Re_{r_0}.

(2) The universality of the velocity defect law of a smooth cylinder under isothermal conditions is also preserved on a rough cylinder irrespective of k^+, and the hydrodynamic function of roughness in the law of the wall for a rough axisymmetric surface does not depend on its transverse curvature when the coordinate $y_R = r_0 \ln (r/r_0)$ is employed.

(3) Formulae are obtained for engineering calculations of heat transfer and friction that interpolate the entire region of transition from the regime without the effect of roughness to the regime with a fully effective roughness and that take into account the variability of physical properties of gas with an increase of the temperature factor up to $\psi = 3$.

(4) The influence of the variability of physical properties on heat transfer increases sharply over the initial stretch where there is a rapid growth in the thicknesses of the hydrodynamic and thermal boundary layers, however, at $x/r_0 \approx 40$ this effect becomes stabilized at a certain level, for each Re_{r_0}.

(5) When the determining quantities are used for $Re_{\delta*}$, which includes the variation of mass velocity across the boundary layer, and $k_w^+ = ku_{*\psi}/v_w$, which takes into account reduced momentum transfer because of the change in the physical properties, experimental data on heat transfer and friction of a heat generating rough cylinder at variable physical properties can be correlated by the same relations as those used at constant physical properties.

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TRANSFERT DE CHALEUR ET FROTTEMENT SUR UN CYLINDRE DANS L'ECOULEMENT TURBULENT, LONGITUDINAL D'UN GAZ A PROPRIETES PHYSIQUES VARIABLES

Résumé—Dans la veine cylindrique d'une soufflerie à grande pression, une recherche expérimentale est conduite sur le transfert de chaleur et le frottement pour un cylindre chauffé, à rugosité rectangulaire, dans un écoulement turbulent gazeux longitudinal avec les domaines de variation suivants: $Re_x = 6\times10^4$–3×10^7, $Re_{r_0} = 2\times10^4$–2×10^5, $k^+ = 5$–200 et $\psi = T_w/T_f = 1-3$. On mesure les profils de vitesse et de température dans la couche limite; leur déformation avec l'accroissement de k^+ est montré, ainsi que l'universalité de la loi d'excès de vitesse et de la fonction hydrodynamique de rugosité et des relations sont présentées pour trouver l'épaisseur de quantité de mouvement δ^{**} dans le cas d'un cylindre rugueux. Des formules sont obtenues pour des calculs d'ingénieur sur le transfert de chaleur et la résistance hydraulique qui interpole l'entière région de transition entre le régime sans effet de rugosité, en tenant compte de la variation des propriétés physiques du gas à travers le facteur de température allant jusqu'à $\psi = 3$.

WÄRMEÜBERGANG UND STRÖMUNGSWIDERSTAND AN EINEM LÄNGS ANGESTRÖMTEN RAUHEN ZYLINDER BEI TURBULENTER GASSTRÖMUNG MIT TEMPERATURABHÄNGIGEN STOFFWERTEN

Zusammenfassung—Im zylindrischen Arbeitsbereich eines Hochdruck-Windkanals werden der Wärmeübergang und der Strömungswiderstand an einem beheizten, längs angeströmten Zylinder mit rechteckförmiger Wandrauhigkeit in einer turbulenten Gasströmung im Bereich $Re_x = 6\times10^4$–3×10^7, $Re_{r_0} = 2\times10^4$–2×10^5, $k^+ = 5$–200 und $\psi = T_w/T_f = 1$–3 untersucht. Die Profile von Strömungsgeschwindigkeit und Temperatur in der Grenzschicht werden gemessen. Die Änderung dieser Profile mit steigendem ψ und k^+ wird gezeigt. Es werden Beziehungen zur Bestimmung der Impulsdicke δ^{**} in der Grenzschicht eines rauhen Zylinders angegeben. Gleichungen zur praktischen Berechnung des Wärmeübergangs sowie des mittleren Strömungswiderstandes werden entwickelt, gültig für das gesamte Übergangsgebiet von den Strömungen ohne Einfluß der Rauhigkeit bis hin zu deren vollem Einfluß. Die Temperaturabhängigkeit der Stoffwerte des Gases wird bis zu einer Temperaturerhöhung von $\psi = 3$ berücksichtigt.

ТЕПЛООТДАЧА И ТРЕНИЕ ЖЕРОХОВАТОГО ЦИЛИНДРА, ПРОДОЛЬНО ОБТЕКАЕМОГО ТУРБУЛЕНТНЫМ ПОТОКОМ ГАЗА С ПЕРЕМЕННЫМИ ФИЗИЧЕСКИМИ СВОЙСТВАМИ

Аннотация—В цилиндрической рабочей части аэродинамической трубы высокого давления в диапазонах $Re_x = 6\times10^4$–3×10^7, $Re_{r_0} = 2\times10^4$–2×10^5, $k^+ = 5$–200 и $\psi = T_w/T_f = 1$–3 экспериментально исследованы теплоотдача и трение тепловыделяющего цилиндра с шероховатостью прямоугольной формы, продольно обтекаемого турбулентным потоком газа. Измерены профили скорости и температуры в пограничном слое, показана их деформация с ростом ψ и k^+, универсальность закона дефекта скорости и гидродинамической функции шероховатости, представлены зависимости для расчета толщины потери импульса δ^{**} в пограничном слое шероховатого цилиндра. Получены формулы для инженерных расчетов теплоотдачи и гидравлического сопротивления, интерполирующие всю переходную область от режима без проявления шероховатости до режима с полным проявлением шероховатости и учитывающие переменность физических свойств газа с ростом температурного фактора до $\psi = 3$.